Implementing superposition in iProver

(System description)

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Corollary:

It's better to run many strategies for a shorter time than one strategy for a long time.

Superposition — generating inferences

$$\text{Superposition} \qquad \frac{l = r \vee C \quad t[s] \doteq u \vee D}{(t[s \mapsto r] \doteq u \vee C \vee D)\theta}$$

where $\theta = \mathrm{mgu}(l, s)$, $l\theta \not\preceq r\theta$, $t\theta \not\preceq u\theta$, and s not a variable,

$$\text{Eq. Resolution} \qquad \frac{l \neq r \vee C}{C\theta}$$

where $\theta = \mathrm{mgu}(l, r)$,

$$\text{Eq. Factoring} \qquad \frac{l = r \vee l' = r' \vee C}{(l = r \vee r \neq r' \vee C)\theta}$$

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Superposition — simplifying inferences

Tautology deletion $\dfrac{\cancel{l \vee \bar{l} \vee C}}{}$ $\dfrac{\cancel{t = t \vee C}}{}$

Syntactic eq. res. $\dfrac{\cancel{t \neq t \vee C}}{C}$

Subsumption $\dfrac{\cancel{C\theta \vee D} \quad C}{}$

Subset subsumption $\dfrac{\cancel{C \vee D} \quad C}{}$

Demodulation $\dfrac{l = r \quad \cancel{C[l\theta]}}{C[l\theta \mapsto r\theta]}, \quad \begin{array}{l} l\theta \succ r\theta \\ \{l\theta = r\theta\} \prec C \end{array}$

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We introduce the simplification rule

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where $l \succ r$, and l occurs outside a maximal side of an equality literal.

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Can be implemented with a hashtable + index of subterms. To normalise a clause, simply traverse bottom-up and look up subterms in the hashtable. To add a new equation, first use-it to reduce equations already kept, then add to the hashtable.

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where $l \to r \in R$, and l occurs outside a maximal side of an equality literal.

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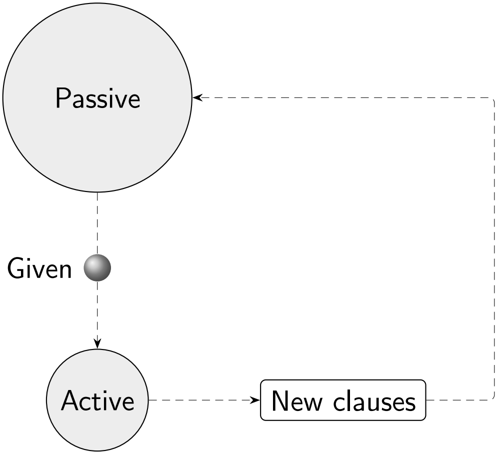
Simplification setup — Given clause loop overview



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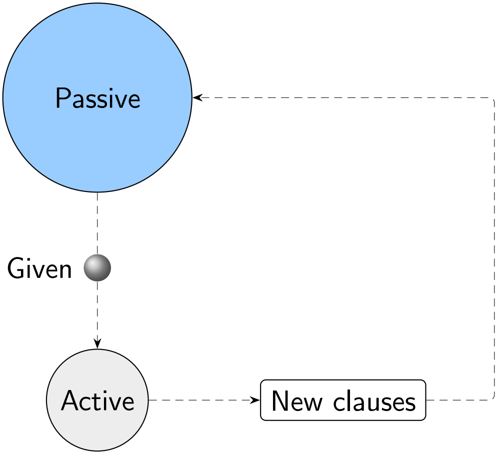

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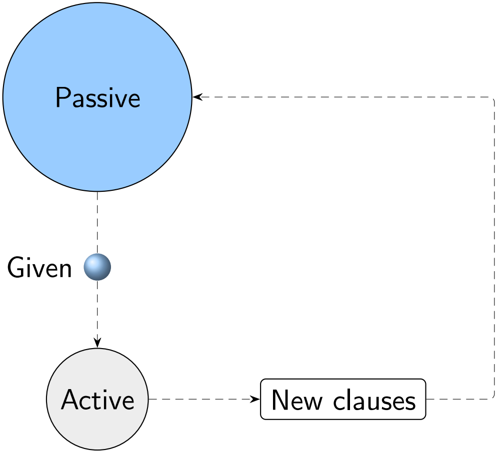

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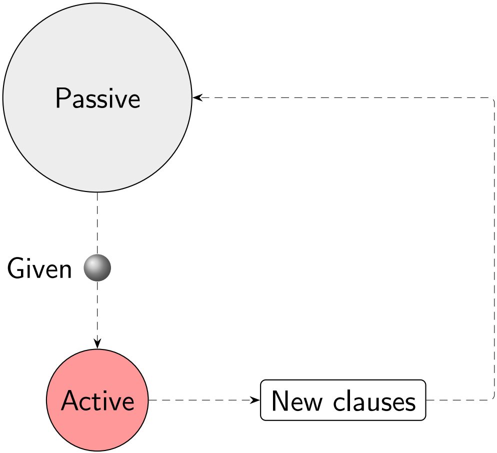

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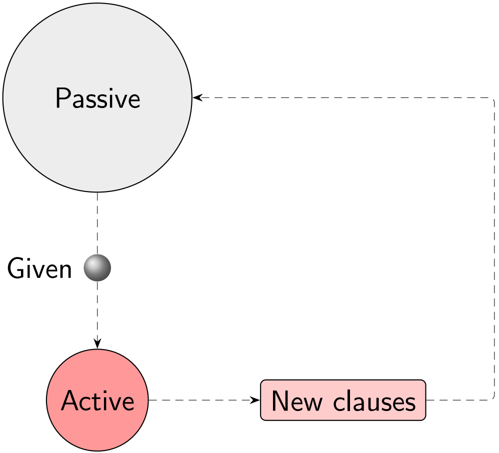

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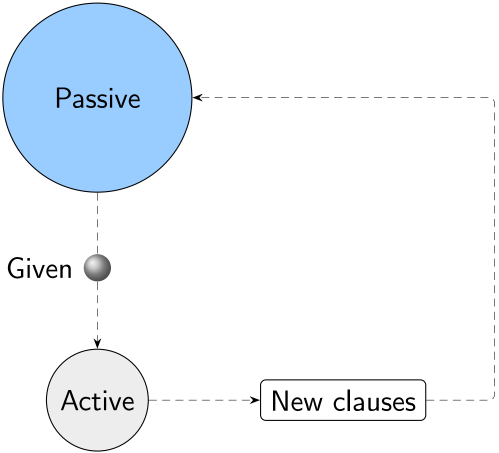

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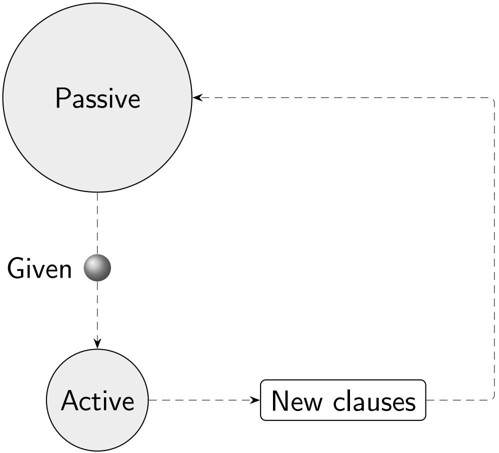

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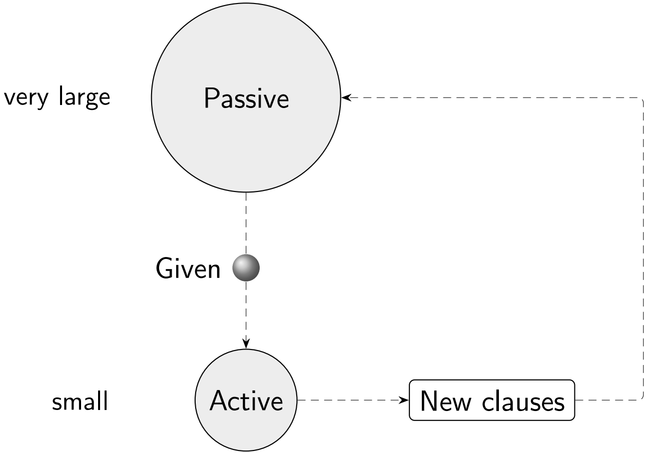

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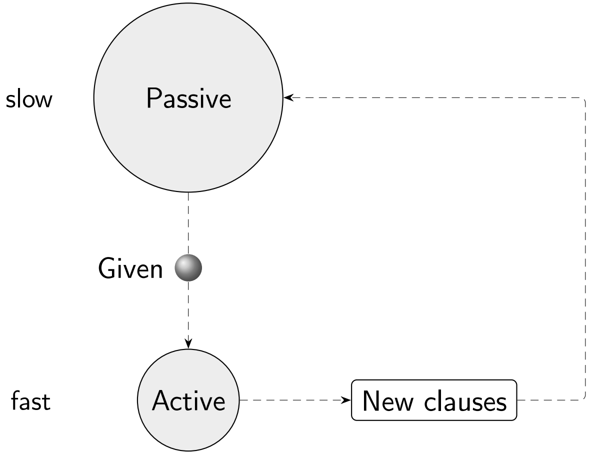

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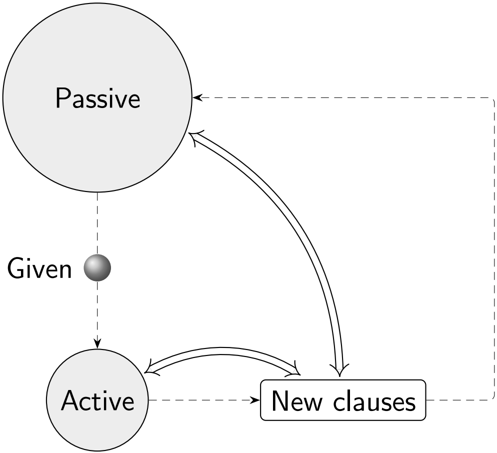
Otter



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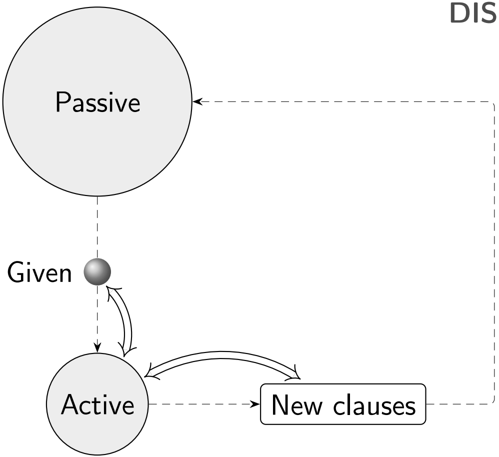
DISCOUNT



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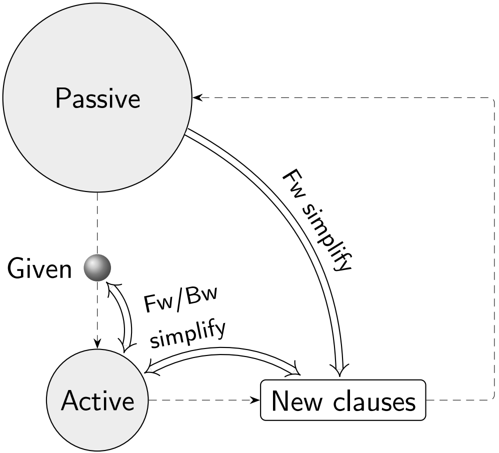

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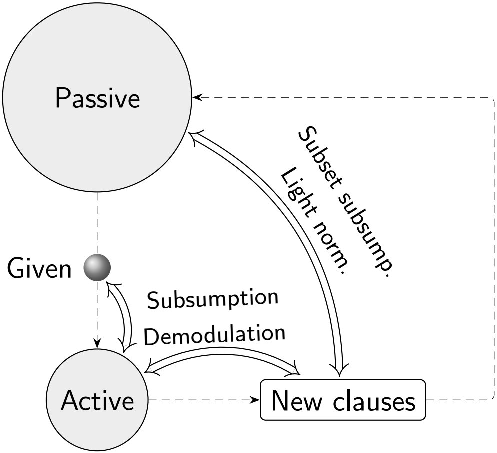

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Hypothesis: it may be useful to keep new clause $\cup$ parents inter-simplified.

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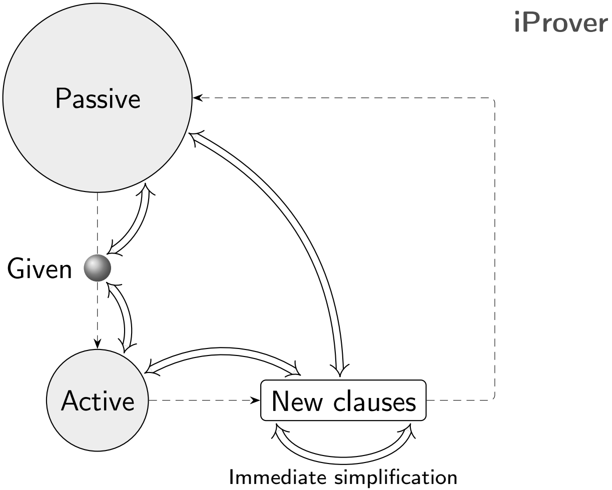

Simplification setup — iProver

```
$ ./iprover --schedule none | grep '--sup'
--sup_indices_passive     [SubsetSubsumption;LightNorm]
--sup_indices_active      [Subsumption;FwDemod;BwDemod]
--sup_indices_immed       [SubsetSubsumption;Subsumption;LightNor...
--sup_indices_input       [SubsetSubsumption;Subsumption;FwDemod;...
--sup_full_triv           [TrivRules;PropSubs]
--sup_full_fw             [FwDemod;ACNormalisation;FwSubsumption;...
--sup_full_bw             [BwDemod]
--sup_immed_triv          [TrivRules]
--sup_immed_fw_main       [FwDemod;ACNormalisation;FwSubsumption]
--sup_immed_bw_main       []
--sup_immed_fw_immed      [FwDemod;FwSubsumption;FwSubsumptionRes]
--sup_immed_bw_immed      [BwDemod;BwSubsumption]
--sup_input_triv          [TrivRules]
--sup_input_fw            [FwDemod;FwSubsumption;FwSubsumptionRes]
--sup_input_bw            [BwDemod;BwSubsumption;BwSubsumptionRes]
```

AC symbols

A symbol f is associative-commutative (AC) iff

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$\implies$ Techniques to handle AC are important.

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- (stably) sorted wrt. the reduction ordering being used.

AC joinability

Theorem. Let R_{AC} be

$$f(x, y) = f(y, x) \tag{1}$$
$$f(x, f(y, z)) = f(f(x, y), z) \tag{2}$$
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if $l = r$ is not an instance of an eq. in R_{AC} and cannot be simplified via a rule in R_{AC}, then if l and r are equal modulo AC then $l = r \vee C$ is a tautology and $l \neq r \vee C$ simplifies to C.

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Tests for AC joinability are cheap!

Semantic AC detection

In general, f is AC if the input problem implies the AC axioms.

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Important to detect AC symbols as soon as possible so we can apply AC reasoning early. We do this in two more ways:

- Detect if AC axioms are derived normally in saturation,
- Check in preprocessing if axioms are implied via fast approximations like ground implication checking using an SMT solver.

Summary

- It is generally better to combine many strategies/options than to run just one.
  - In particular, instantiation + superposition performs better than just instantiation or just superposition.
- Huge freedom in choosing how to do simplifications, but no clear path.
  - Work on hyperparameter optimisation may help here.
- "Immediate simplification" can block many redundant generating inferences, and is relatively inexpensive.
- AC reasoning speeds up many problems (axioms found in 12% of TPTP (incl. by semantic detection)).