Experimenting with superposition in iProver

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The University of Manchester

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Corollary:

It's better to run many strategies for a little time than one strategy for a long time.

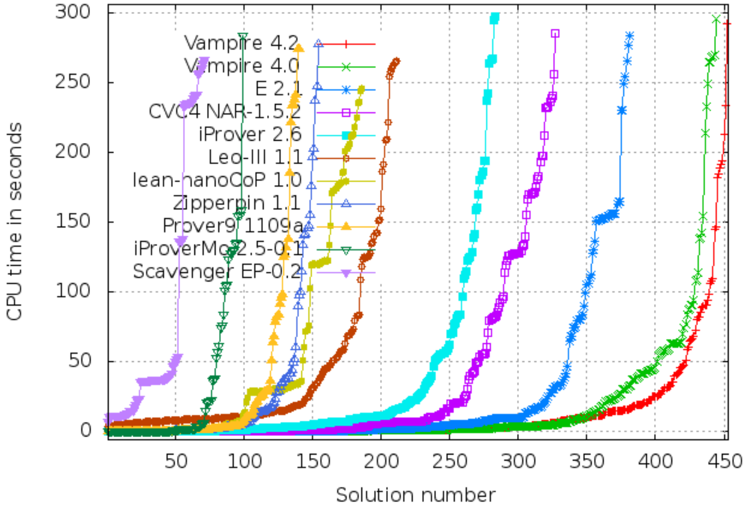

Figure: Performance graph for provers entered CASC-26/FOF.

Superposition

$$\text{Superposition} \qquad \frac{l = r \vee C \quad t[s] \doteq u \vee D}{(t[s \mapsto r] \doteq u \vee C \vee D)\theta}$$

where $\theta = \mathrm{mgu}(l, s)$, $l\theta \not\preceq r\theta$, $t\theta \not\preceq u\theta$, and s not a variable,

$$\text{Eq. Resolution} \qquad \frac{l \neq r \vee C}{C\theta}$$

where $\theta = \mathrm{mgu}(l, r)$,

$$\text{Eq. Factoring} \qquad \frac{l = r \vee l' = r' \vee C}{(l = r \vee r \neq r' \vee C)\theta}$$

where $\theta = \mathrm{mgu}(l, l')$, $l\theta \not\preceq r\theta$ and $r\theta \not\preceq r'\theta$.

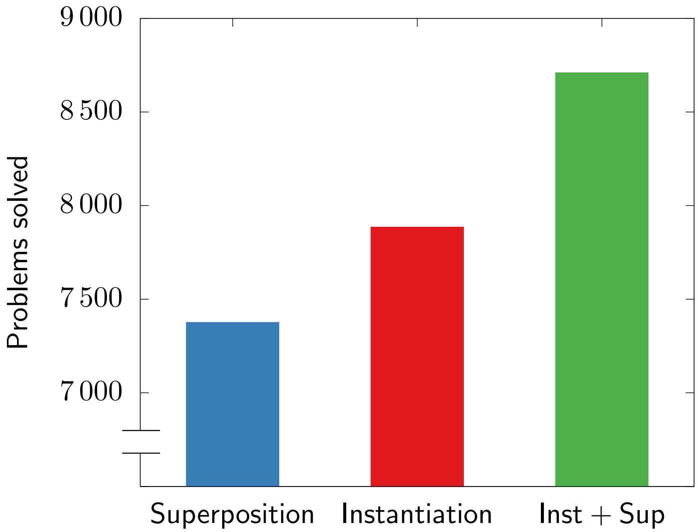

Figure: Number of problems solved over TPTP-v7.2.0, in less than 300 s.

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> Simplifying inferences are key. . .
> but we can't spend too much time on them!

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Tautology deletion

$$\frac{\cancel{l \vee \bar{l} \vee C}}{} \qquad \frac{\cancel{t = t \vee C}}{}$$

Syntactic eq. res.

$$\frac{\cancel{t \neq t \vee C}}{C}$$

Subsumption

$$\frac{\cancel{C\theta \vee D} \quad C}{}$$

Subset subsumption

$$\frac{\cancel{C \vee D} \quad C}{}$$

Demodulation

$$\frac{l = r \quad \cancel{C[l\theta]}}{C[l\theta \mapsto r\theta]}, \quad \begin{array}{l} l\theta \succ r\theta \\ \{l\theta = r\theta\} \prec C \end{array}$$

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Light normalisation
$$\frac{l = r \quad \cancel{C[l]}}{C[l \mapsto r]}, \quad l \succ r$$

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We introduce the simplification rule

$$\text{Light normalisation} \qquad \frac{l = r \quad \cancel{C[l]}}{C[l \mapsto r]}$$

where $l \succ r$, and l occurs outside a maximal side of an equality literal.

While a restricted case of demodulation, it's also much faster.

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- Long demodulation chains are done in 1 step.

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- which clauses to add to which indices,
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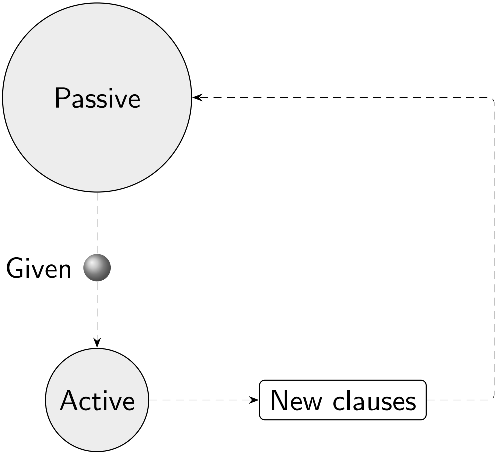

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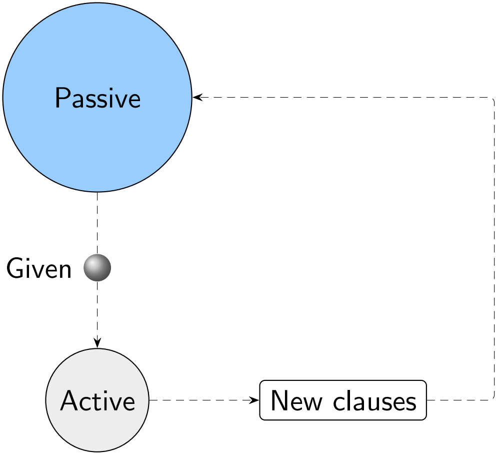

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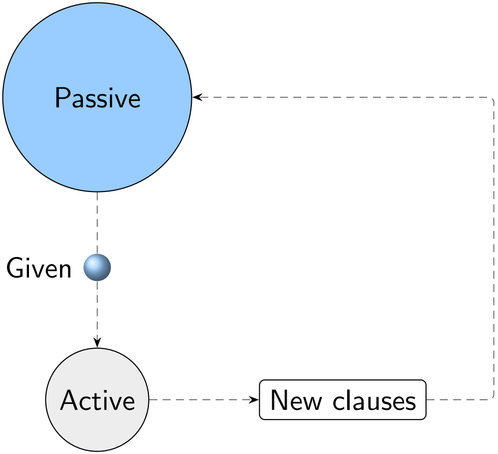

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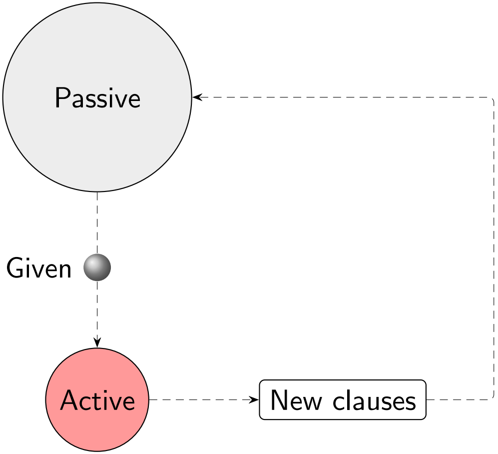

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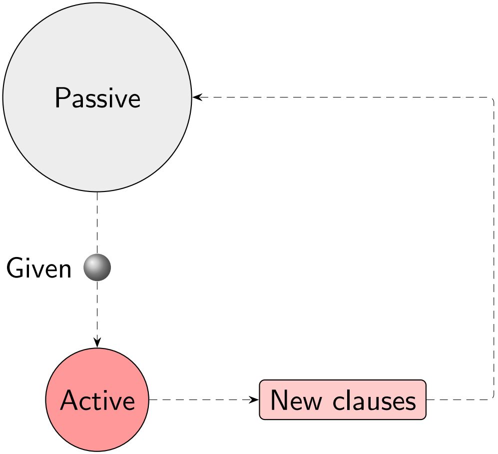

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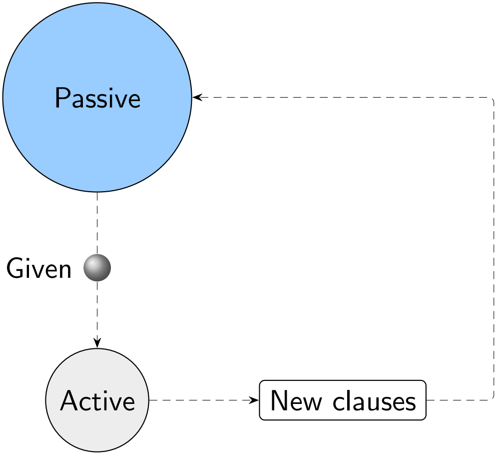

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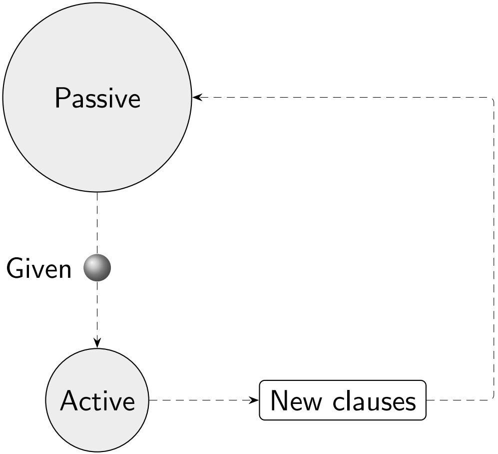

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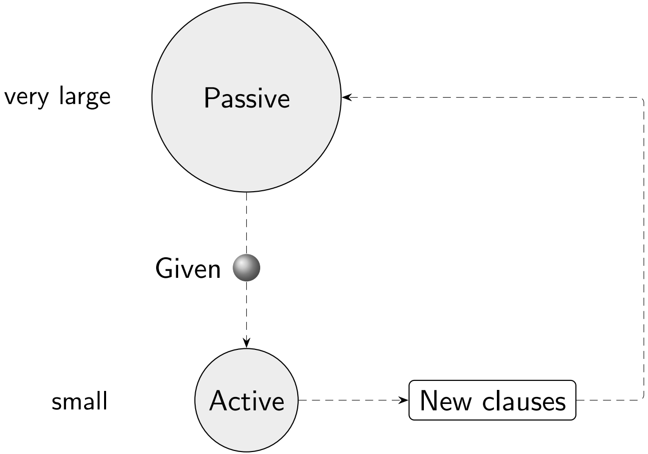

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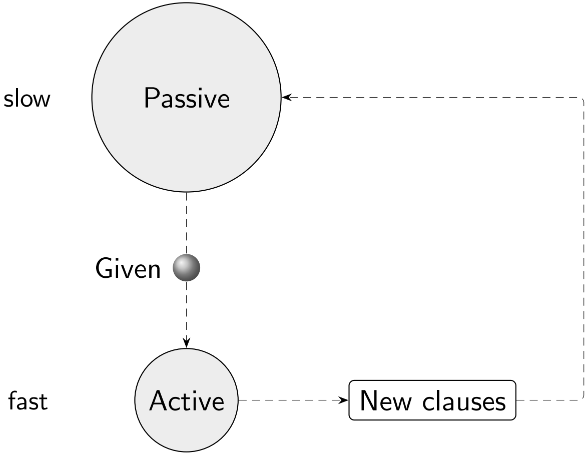

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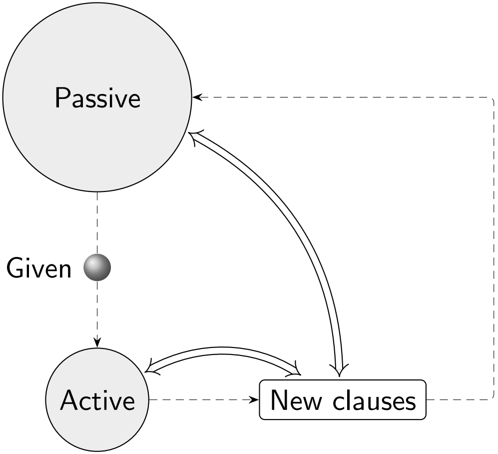
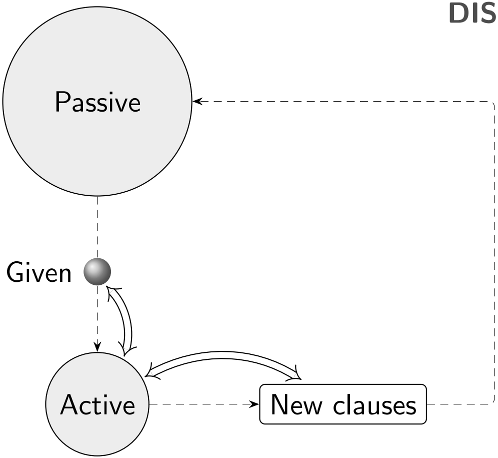


Simplification setup

DISCOUNT



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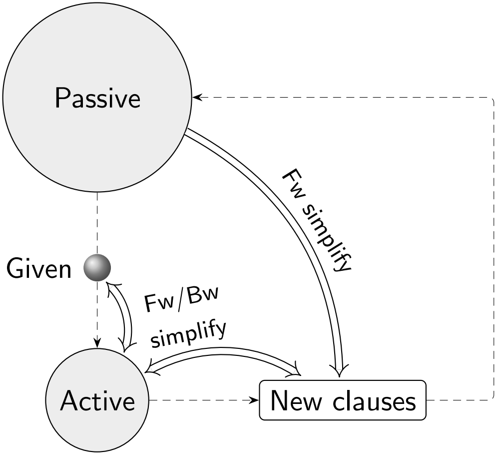
iProver



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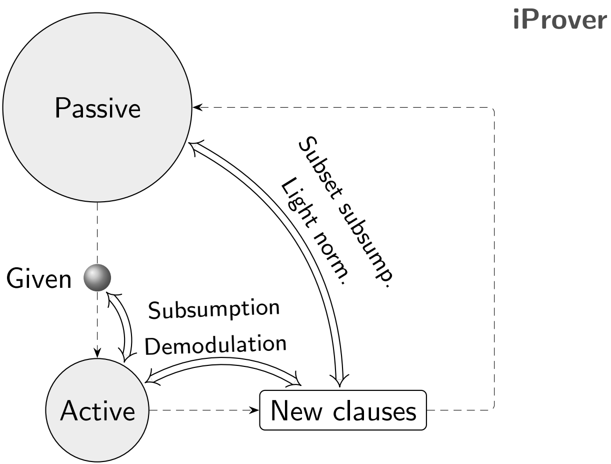

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Hypothesis: it may be useful to keep new clause $\cup$ parents inter-simplified.

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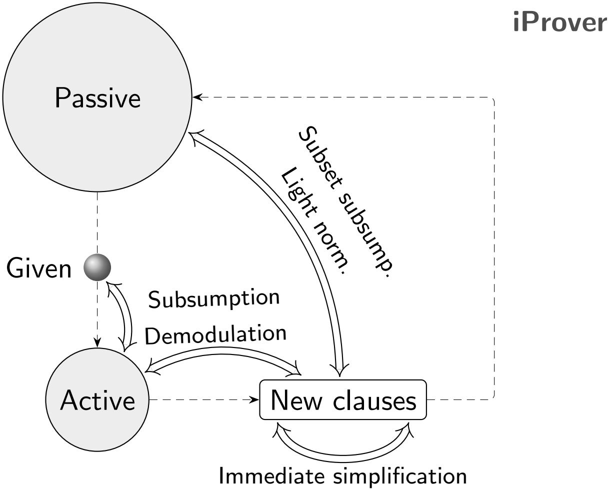

Simplification setup — iProver

```
$ ./iprover --schedule none | grep '--sup_'
--sup_indices_passive     [SubsetSubsumption]
--sup_indices_active      [Subsumption;LightNormNoReduce;FwDemod;...
--sup_indices_immed       [SubsetSubsumption;Subsumption;LightNor...
--sup_indices_input       [SubsetSubsumption;Subsumption;LightNor...
--sup_light_triv          [TrivRules]
--sup_light_fw            [FwLightNorm]
--sup_light_bw            []
--sup_full_triv           [TrivRules;PropSubs]
--sup_full_fw             [FwDemodLightNormLoopTriv;FwSubsumption...
--sup_full_bw             [BwDemod]
--sup_immed_triv          [TrivRules]
--sup_immed_fw_main       [FwDemodLightNormLoopTriv;FwSubsumption...
--sup_immed_fw_immed      [FwDemodLightNormLoopTriv;FwSubsumption...
--sup_immed_bw_main       []
--sup_immed_bw_immed      [BwDemod;BwSubsumption;BwSubsumptionRes...
--sup_input_triv          [TrivRules]
--sup_input_fw            [FwDemodLightNormLoopTriv;FwSubsumption...
--sup_input_bw            [BwDemod;BwSubsumption;BwSubsumptionRes]
```

Summary

- It is (generally) better to combine many strategies/options than to run just one.
  - Instantiation + superposition is better than just instantiation or superposition.
- Applying simplification rules is crucial for performance. But spending too much time on them may hurt more than help.
- Huge freedom in choosing how to do them, but no clear path.
  - Work on hyperparameter optimisation may help here.
- "Immediate simplification" may block many redundant generating inferences, and is relatively inexpensive.